UNIT-II PERMUTATION & COMBINATION

COUNTING PRINCIPLE-

When we perform 2 or more actions with specified no of options (say m & n), then,

For all the actions together we should MULTIPLY the options= mxn

Whereas,

For any one action at a time we should ADD the options= m+n

We can apply the counting principle as below:

Suppose action A can be done in 5 different ways & action B inn 4 different ways then,

Both A & B together can be done in = 5x4=20 possible ways

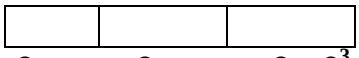
Whereas,

Any one of them i.e. action A or action B can be done in = 5+4=9 possible ways

Remark: The counting principle can be applied to mor than 2 actions also shown as below.

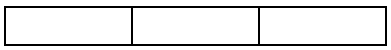
Example: Suppose we want to write a 3 digit number(3 actions) using digits 1,3,4,5,6,7,8 & 9. This is the arrangement of 8 digits in 3 boxes, which can be done as

With repetition



8 x 8 x 8 = 8^3 =512 ways

without repetition



8 x 7 x 6 = 336 ways

This is called arrangement is called as Permutation of 8 taken 3 at a time.

Hence, permutation of 8 taken 3 at a time means, putting 8 objects in 3 boxes with or without repetition

For repetition answer is 8^3 & without repetition answer is 8x7x6

Permutation without repetition is written as $^8P_3$= 8x7x6

On the other hand, when we want to select 3 items out of 8 this is called Combination.

Hence, combination of n taken r at a time gives no of options to select r out of n. it is written as ^nC_r

Hence,

$^5P_3$**= arranging 5 on 3 places, like assigning 3 classes to 5 teachers**

Calculation formula;

$$^{n}P_r = \frac{n!}{(n-r)!} \qquad ^{n}C_r = \frac{n!}{r!(n-r)!}$$

These calculations can also be done easily as,

$^{8}P_3$= 8x7x6 (3 numbers in reverse order) & $^{8}C_3 = \frac{8x7x6}{3x2x1}$

Illustrations: Evaluate the following,

i) $^{5}P_3 + ^{7}P_2 = 5x4x3 + 7x6 = 120+42=162$

ii) $^{8}P_3 + ^{6}P_4 = 8x7x6+6x5x4 = 456$

iii) $^{8}C_4 + ^{7}C_3 = \frac{8x7x6x5}{4x3x2x1} + \frac{7x6x5}{3x2x1} = 70+35=105$

iv) $^{8}C_6 + ^{8}C_7 = \frac{8x7x6x5x4x3}{6x5x4x3x2x1} + \frac{8x7x6x5x4x3x2}{7x6x5x4x3x2x1} = 28+8=36$

Note- Students can remember and practice this easy trick for calculations

ILLUSTRATIVE EXAMPLES

1. In how many possible ways 4 books on Mathematics & 3 books on English can be arranged in a row so that,
   - Books of same subjects are always together
   - Only English books are together
   - English books are never together

Solution: **Remark:** When books of same subjects are together count them ONE group.

- When subject books are together, we count 2 bundles of 2 subjects, these can be arranged in $^{2}P_2$ways
  Also in each bundle 3 English books can be arranged in $^{3}P_3$ways & 4 Maths books in $^{4}P_4$ways.
  Hence we have, by counting principle

  No of arrangements = $^{2}P_2 X ^{3}P_3 X ^{4}P_4$= 5! X 3! X 4!

- When only English books are together, we have
  1 English (bundle) +4Maths=5 to be arranged in 5 boxes in $^{5}P_5$ways

  Also 3 English books in the bundle can be arranged in $^{3}P_3$ways

Now by counting principle

Total no of arrangements = ${}^5P_5 X {}^3P_3 = 5!\ X\ 3!$

- When English books are never together, we put at least 1 Maths book between two English books, hence a possible arrangement is

E	M	E	M	E	M	E	M	E

i,e, English book can be kept between two Maths books and also at the end positions. Therefore there are 5 places to put 3 English books, which can be done in ${}^5P_3$ ways.

Also 4 Maths books can be arranged in ${}^4P_4$ ways. Hence by counting principle,

Total no of arrangements are ${}^5P_3 \times {}^4P_4 = 5x4x3x4!$ ways.

2. In how many possible 3 digits numbers can be generated from the digits 1,2,4,5,6,7,9 when,
   - Digits are allowed to repeat
   - Not allowed to repeat
   - Number written is less than 500 without repetition

   Solution:

   We have total 7 digits to be put in 3 boxes. This can be done in,

   i. $7x7x7 = 343$ when repetition is allowed &
   ii. ${}^7P_3$ or 7x6x5 possible ways by counting principle
   iii. When number written is less than 500 without repetition, first digit must be less than 5 always (i.e.1,2 or 4). Hence,

   Total possible arrangements= 3X6X5=90 by counting principle

3. Out of 5Boys, 4 Girls & 3 Teachers a committee of 5 to be formed. In how many possible ways can this be done when,
   - It includes 2Boys 2 Girls & 1 Teacher
   - 2Boys+ 2 Girls+1 Teacher with 1 particular Girl is must
   - 2Boys 2 Girls & 1 Teacher with 1 particular Girl is not available for selection

   Solution: We have 5+4+3= 12 people

   - Now in the committee of 5(in2+2+1)

     i.e. 2B out of 5 & 2G out of 4 & 1T out of 3

No of options= ${}^5C_2 \times {}^4C_2 \times {}^3C_2$ possible committees

- In (2+2+1) when particular Girl is must. We need to select only 1 girl from 3

  i.e. 2B out of 5 & 1G out of 3 & 1T out of 3

  No of options = ${}^5C_2 \times {}^3C_1 \times {}^3C_1$ possible committees

- In (2+2+1) when particular Girl is out. We need to select 2 girl from remaining 3

  i.e. 2B out of 5 & 2G out of 3 & 1T out of 3

  No of options = ${}^5C_2 \times {}^3C_2 \times {}^3C_1$ possible committees

Remark: Students should note carefully the options when 1 particular is must & when it is out.

4. In how many possible the letters in the word FATHER be arranged so that,

   a) All the vowels are always together b) they are not together

Solution: The word FATHER has 2 vowels & 4 consonants

a) Now when both vowels are together we count them 1 & so the 4 consonants hence 2 groups of 2V&4C. Therefore No of arrangements= 2!x2!x4!

b) When 2 vowels are not together, the possible arrangement is

V	C	V	C	V	C	V	C	V

Hence there are 5 places for 2 vowels which can be filled in ${}^5P_2$ ways.

Whereas, 4 consonants can be placed in ${}^4P_4 = 4!$ ways.

Therefore total arrangements= ${}^5P_2 \times 4!$

5. Five books on Mathematics, 4 books on English & 3 books on History are to be put in a shelf in a row. In how many possible ways can this be done so that,

   - Books of same subjects are always together
   - Only English books are together
   - No two Mathematics are together

Solution: There are 5 Mathematics, 4 English & 3 History books.

- When, all subject books are together there are 3 bundles of 5,4&3 books. This can be arranged in 3!x5!4!x3! ways

- When 3 English books are together, we have 5+4+1(bundle of English)=10 books to arrange with 3 English books together. This can be done in 10!x3! possible ways
- Now when no two Mathematics books are together, between 2 Maths books there will be a book of other (either History/English) subject. Hence the possible arrangement could be,

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
M	O	M	O	M	O	M	O	M	O	M	O	M	O	M

Therefore, there are 8 boxes to put 5 Maths books & 7 boxes to put 7 books of other subjects. This can be done in, ${}^{8}P_{5}$ X7!.

6. A box contains 6 Green & 5 Red balls. A pair of balls is drawn at random. Find the no of possible selections so that,
   i) Both the balls are of same color.
   ii) They are of different colors
   iii) Only red balls are drawn

Solution: In the box there are 6Green & 5 Red balls.

i) When a pair of same colour ball is drawn (selected) it could be 2 Green OR 2 Red. This can be done in combination 2Green out of 6Green OR 2 Red out of 5Red with

No of options = ${}^{6}C_{2}$ + ${}^{5}C_{2}$ = $\frac{6x5}{2\,x1}+\frac{5x4}{2\,x1}$ = 15+10=25 pairs

ii) When a pair of different colour is drawn (selected) it could be 1 Green & 1 Red. This can be done in combination

1Green out of 6Green & 1 Red out of 5Red with

No of options = ${}^{6}C_{1}$ x ${}^{5}C_{1}$ = 6x5=30 pairs

iii) When a pair of Red colour ball is drawn (selected) it could be 2 Red out 5 Red. This can be done with options, ${}^{5}C_{2}$= 10 options.

7. In how many possible ways 3 cards can be drawn from the pack of 52 cards so that,
   i) all 3 are Ace cards;
   ii) there are two kings and one queen
   iii) cards are of same suit

Solution:

i) When all 3 are Ace cards;

There are 4 Ace cards of them 3 can be drawn in ${}^4C_3$= 4 possible options.

ii) When there are two kings and one queen

In the pack there are 4 Kings & 4 Queens, hence 2 Kings & 1 Quuen card can be drawn with ${}^4C_2$X ${}^4C_1$(using counting principle)= 6x4= 24 options.

iii) When the cards are of same suit; there are 4 suits of 13 cards each. Cards of one suit can be drawn with ${}^{13}C_3$= 286 options. And by counting principle for cards of same suit the no of options are cards

${}^{13}C_3$+ ${}^{13}C_3$+ ${}^{13}C_3$+${}^{13}C_3$= 4x286=1144 possible options.

EXAMPLES FOR PRACTICE

1. Evaluate the following,

${}^5P_3$ +${}^7P_2$ ii) ${}^8P_3$ +${}^6P_4$ iii) ${}^{10}P_8$ +${}^{10}P_7$

2. In how many possible the letters in the word ATTITUDE be arranged so that,
   - All the vowels are always together
3. All the consonants are togetherFour books on PHYSICS 3 books on CHEMISTRY & 2 books on BIOLOGY are to be put in a shelf in a row. In how many possible ways can this be done so that,
   - Books of same subjects are always together
   - Only BIOLOGY books are together
   - BIOLOGY books are at end position
4. Six boys & 2 Girls are to stand in a row for a group photo. How possible ways they can have a photo so that,
   - 2 Girls always stand together
   - They do not stand together
   - They stand at the end position?
5. From the digits 1,2,5,6,8 & 9 a 3 digits number is to be formed. In how many possible this can be done so that,
   - No digit is repeated

- Digits are allowed to repeat
- Only even number without repeated is formed.

6. In how many possible ways 2 balls can be drawn out of 15 balls?
7. Find the no of possible ways to draw a pair of cards from the pack of 52 playing cards.
8. A box contains 6 Green & 5 Red balls, a pair of balls is drawn at random. Find the no of possible selections so that,
   - Both the balls are of same colours.
   - They are of different colours
9. Out of 6 Batsmen, 5 Bowlers & 3 Wicketkeeper a Team of 11 players is to be formed. How many ways can this be done so as to include,
   - 5 Batsmen, 4 Bowlers & 2 Wicketkeeper
   - 6 Batsmen, & at most 1 Wicketkeeper
10. In how many possible ways 3 cards can be drawn from the pack of 52 cards so that, i) all 3 are Ace cards; ii) there are two kings and one queen. iii) cards are of same suit.